HYPER VERSION

Received: *June 23, 1999*, Accepted: *July 23, 1999*

# Multi-strings on $AdS_3 \times S^3$ from matrix string theory

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Abstract: We analyze the Coulomb branch of Matrix string theory in the presence of NS5-branes. If we regard the components of U(1) gauge fields as the dualized longitudinal coordinates, we obtain the symmetric product of $AdS_3 \times S^3 \times \mathbf{R}^4$ as the geometry of Coulomb branch. We observe that the absence or presence of the non-zero electric flux determines whether the string propagates in bulk as an ordinary closed string or is forced to live near the boundary. We further discuss the issues of the physical spectrum from the viewpoint of Matrix string theory. We show that the twisted sectors of CFT on the symmetric orbifold, which correspond to glued strings, turn out to yield many chiral primaries that were hitherto considered to be missing. We also comment on the threshold energy in Liouville sector where continuous spectrum begins.

Keywords: M(atrix) Theories, Models of Quantum Gravity, Cosmology of Theories beyond the SM.

JHEP07(1999)027

## Contents





## 1. Introduction

The theory of gravity or string theory on three-dimensional anti de-Sitter space ($AdS_3$) recently has been a topic of considerable interest. It has been understood for a long time that the gravitational degrees of freedom in the bulk of $AdS_3$ may be described by a conformal field theory on the boundary. After Maldacena presented the celebrated conjecture on the duality between the gravity on the $AdS$ space and CFT on its boundary [1], the case of $AdS_3$ achieved particular interest, as we know CFT in two dimensions best. It may be possible to prove the Maldacena's conjecture in more detail in the case of $\mathrm{AdS}_3/\mathrm{CFT}_2$. Much work has been done along this path [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

In string theory, the $AdS_3$ space arises as the near-horizon geometry of the bound states of D1/D5-branes. We take its S-dual and consider the string theory on an NS1/NS5-brane background, in order to avoid the difficulty in considering string theories on RR backgrounds. The duality between string theory on $AdS_3$ and a certain boundary CFT has been discussed extensively. The generators of boundary superconformal algebras were directly constructed out of vertex operators on the superstring worldsheet [2]. There have been attempts to study the spectrum of chiral primaries and observe the correspondence to some extent [5, 7].

There have been further attempts to realize the boundary CFT Ward identities in which the $AdS_3$ string theory was treated in a different way [6]. Although there was some confusion between these two approaches, some recent works [11, 12] put these concepts in order. Now it turns out that the string theory on $AdS_3$ has two distinct sectors corresponding to two different configurations of the worldsheet. They are called as the "short string" sector and the "long string" sector. The long strings are forced to live near the boundary with some fixed winding number, while the short strings can have arbitrary configurations of worldsheet and are free to propagate in bulk. The two sectors lead to two different explanations for the origin of the central charge of the boundary CFT, as was discussed in [12]. The main purpose of the present paper is to propose a unified framework including both the short and long string sectors, based on Matrix string theory [14].

One of the unresolved problems regarding the spectrum of chiral primaries is the mismatch of its upper bound. In the string theory on $AdS$ space it is of order $\sim k$, where $k$ is the number of 5-branes. However, it is argued that the bound becomes of order $\sim pk$ in the boundary CFT, where $p$ is the number of 1-branes. As was stated in our previous paper [7], it is natural to think that this discrepancy is due to the fact that we are only considering the theory of a single string on $AdS_3$. We expect that the multi-string system based on Matrix string theory may solve this inconsistency. In this paper we show that this is indeed the case.

This paper is organized as follows. We begin with the analysis of the Coulomb branch of Matrix string theory in the presence of NS5-branes. We are then lead to the sigma model on the moduli space of the Coulomb branch, which is identified as the second quantized string theory on $AdS_3 \times S^3 \times \mathbf{R}^4$. Twisted sectors of this orbifold CFT correspond to glued strings, which are strings made from some fundamental strings glued together[1]. The long/short string sectors are distinguished by the presence/absence of non-zero electric flux on the worldsheet.

We shall also discuss the spectrum of chiral primaries from the viewpoint of Matrix string theory, especially the issue of missing chiral primaries and the threshold for continuous spectrum pointed out in [12].

Throughout this paper we denote NS5-brane charge as $k$ and NS1-brane charge as $p$, following the convention of [2].

## 2. Analysis of gauge theory

Matrix string theory of type IIA superstring [14] is defined as the large $N$ limit of $\mathcal{N} = (8,8)$ U($N$) SYM theory in two dimensions. To incorporate $k$ longitudinal NS5-branes extending along the 016789-directions, we add $k$ hypermultiplets belonging to the fundamental representation of U($N$). In $\mathcal{N} = (4,4)$ language the system consists of a U($N$) vector multiplet and hypermultiplets belonging to one adjoint and

[1] Hereafter we shall use the term "*glued string*" for strings which are made by gluing some fundamental strings together. We will keep the term "*long string*" for different objects, which appear in [11, 12].

$k$ fundamental representations of U($N$). In the Coulomb branch the gauge group is generically broken down to $\mathrm{U}(1)^N$. The massless scalar fields parametrizing the moduli space are $N$ abelian vectormultiplets, which correspond to the string coordinates along the directions transverse to the NS5-branes (2345 directions), and the $N$ neutral hypermultiplets corresponding to the coordinates along 6789 directions. Usually the 6789 directions are compactified on $T^4$ or $K3$. But in this article we would like to ignore the subtleties concerning the compactification of Matrix string theory and focus mainly on the six-dimensional part.

One can obtain the exact metric of moduli space by one-loop analysis. This was done in [15, 16] for the case of $N = 1$, and similar calculation works also in the case of generic $N$.

In $\mathcal{N} = (2,2)$ terminology, an abelian vector multiplet consists of a chiral and a twisted-chiral multiplets, $(\Phi, \Sigma)$. We can write down the most generic action for these multiplets as an integral of a function $K(\Phi, \bar{\Phi}, \Sigma, \bar{\Sigma})$ over superspace [17]. The condition for $K$ to give a $\mathcal{N} = (4,4)$ supersymmetric gauge theory of vectormultiplets is given by

$$K_{\Phi^i \bar{\Phi}^j} + K_{\Sigma^i \bar{\Sigma}^j} = 0\,, \qquad K_{\Phi^i \bar{\Phi}^j} - K_{\Phi^j \bar{\Phi}^i} = 0\,. \tag{2.1}$$

The metric and $B$ field on the target space are given by

$$ds^2 = K_{\Phi^i \bar{\Phi}^j}\, d\Phi^i\, d\bar{\Phi}^j - K_{\Sigma^i \bar{\Sigma}^j}\, d\Sigma^i\, d\bar{\Sigma}^j\,, \tag{2.2}$$

$$B = -\frac{1}{4}\Big(K_{\Phi^i \Sigma^j}\, d\Phi^i\, d\Sigma^j - K_{\Phi^i \bar{\Sigma}^j}\, d\Phi^i\, d\bar{\Sigma}^j - K_{\bar{\Phi}^i \Sigma^j}\, d\bar{\Phi}^i\, d\Sigma^j + K_{\bar{\Phi}^i \bar{\Sigma}^j}\, d\bar{\Phi}^i\, d\bar{\Sigma}^j\Big)\,. \tag{2.3}$$

Taking Spin(4) and permutation symmetry into account, we can conclude that the most generic form for $K$ is given by

$$K = \sum_i K_i\,, \tag{2.4}$$

$$K_i = a\Big(\Phi^i \bar{\Phi}^i - \Sigma^i \bar{\Sigma}^i\Big) + b\Bigg(\ln \Phi^i \ln \bar{\Phi}^i - \int^{(\Sigma^i \bar{\Sigma}^i)/(\Phi^i \bar{\Phi}^i)} \frac{d\xi}{\xi} \ln(1+\xi)\Bigg)\,. \tag{2.5}$$

The terms with coefficients $a$ and $b$ are tree and one-loop contributions, respectively, and we can fix them as $a = 1/(2g^2)$ and $b = k/(4\pi)$.

We would like to note here that the one-loop contributions to the effective action arise only from the fundamental hypermultiplets, so that the resultant contribution is proportional to $k$. The one-loop contribution of adjoint fields cancel each other. This is to be expected, because the system has a larger supersymmetry (with 16 supercharges) when the fundamental hypermultiplets are absent.

The effective action has the following bosonic part

$$
\begin{aligned}
S &= \sum_i S_i \,, \\
S_i &= -\int d^2x \left(\frac{1}{g^2} + \frac{k}{2\pi y_{(i)}^2}\right)\left[\frac{1}{2}\,\partial_\mu y_{(i)}^p\,\partial^\mu y_{(i)}^p + \frac{1}{4} f_{(i)\mu\nu} f_{(i)}^{\mu\nu}\right] + \\
&\quad + \int_{\mathcal{B}} \frac{k}{6\pi y_{(i)}^4}\,\epsilon_{pqrs}\, dy_{(i)}^p\, dy_{(i)}^q\, dy_{(i)}^r y_{(i)}^s
\end{aligned} \tag{2.6}
$$

$$
i = 1, \ldots, N\,, \qquad \mu, \nu = 0, 1\,, \qquad p, q, r, s = 2, 3, 4, 5\,.
$$

Here $y_{(i)}^p$ and $f_{(i)\mu\nu}$ are the four scalars and the U(1) field strength of the $i$-th abelian vectormultiplet. $\mathcal{B}$ is some open three-dimensional space whose boundary is the worldsheet of Matrix string. In Maldacena's near horizon limit or the weak coupling limit of IIA superstring theory, we send $g^{-2} \equiv g_s^2 l_s^2$ to zero and drop the tree term from the action. Then the metric of the target space is no longer asymptotically flat. The four-dimensional spatial directions now have a tube metric, and we expect to obtain the $AdS_3 \times S^3$ geometry by incorporating the longitudinal directions. After changing the coordinates from $y_{(i)}^p$ to the radial coordinates $y_{(i)}$ and the SU(2) group elements, the action becomes

$$
S_i = -\frac{k}{2\pi}\int_{\mathcal{M}} d^2x \left[\frac{1}{2y_{(i)}^2}\,\partial_\mu y_{(i)}\,\partial^\mu y_{(i)} + \frac{1}{4y_{(i)}^2} f_{(i)\mu\nu} f_{(i)}^{\mu\nu}\right] + \Big(\text{SU(2)WZW with level } k\Big)_i \,. \tag{2.7}
$$

The full supersymmetric extension of this action is then

$$
S_i = -\frac{k}{4\pi}\int_{\mathcal{M}} d^2x\, d^2\theta\, Y_{(i)}^{-2} DY_{(i)} DY_{(i)} + \Big(\text{SU(2)SWZW with level } k\Big)_i \,. \tag{2.8}
$$

Here $Y_{(i)}$ is an $\mathcal{N} = (1,1)$ superfield which has $y_{(i)}$ as the lowest component. Its $\theta$-expansion reads

$$
Y_{(i)} = y_{(i)} + i\theta\psi_{(i)} + \frac{i}{4}\,\theta\theta\epsilon^{\mu\nu} f_{(i)\mu\nu}\,.
$$

Note that the level of the bosonic part of SWZW in eq. (2.8) should be shifted to $k-2$ by means of a chiral rotation so as to make the fermionic fields free, as was discussed in [16, 18, 19]. Hence the level of the total current, including the fermionic contribution, should remain $k$.

## 3. Short strings and long strings

Various interesting phenomena occur when we compactify the spatial direction on $S^1$. Although the gauge field in two dimensions has no dynamics, it carries some topological informations, according to which we can classify the sectors of Matrix string theory.



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Various sectors can be labeled by the periodicity conditions along $S^1$. As was explained in the original papers on Matrix string theory [14], we can twist the periodicity of fields which take values in the Cartan subgroup of U($N$) by elements of the Weyl group, namely, the $N$-th permutation group. The twisted sectors are pictorially understood as glued strings, or some strings mutually glued together into a single string. For example, there is a sector which is labeled by the $N$-th cyclic permutation. This sector is interpreted as the sector where $N$ fundamental strings are joined together to form a glued string. As we shall explain later, the glued strings constructed in this way are accountable for the missing chiral primaries.

Another interesting phenomenon is the presence of non-zero U(1) electric flux. In the flat background with no NS5-brane charge, this flux describes the existence of D0-branes [14], and in the T-dualized framework it describes the $(p, q)$-string sectors [20, 21]. However, as our discussion below is based on the effective action eq. (2.8), the presence of electric flux may have a different interpretation. We shall show below that the string becomes "long" if there is non-zero electric flux on its worldsheet.

To simplify the problem, we focus on the case of a single string (the case $N = 1$) for the time being. To see the implication of non-zero electric flux, let us write down the relevant terms in the effective action below

$$S = -\frac{k}{4\pi}\int_\Sigma d^2x\left(\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\,e^{-2\phi}f^{\mu\nu}f_{\mu\nu}\right) + \cdots . \tag{3.1}$$

Here the "Liouville field" $\phi$ is defined as $y = e^\phi$ and the worldsheet $\Sigma$ is a cylinder or the two-punctured sphere. We can quantize the system canonically in the $A_0 = 0$ gauge. The only dynamical variable is the Wilson line $U = \exp\left(\oint A_1\,dx\right)$. $\psi(U) \sim U^n$ is the eigenfunction of the electric field strength $\Pi \equiv e^{-2\phi}E$, which is the canonical conjugate of the Wilson line. Note that the eigenvalue is quantized due to the periodicity of the Wilson line. When there is non-zero electric flux, it yields the following contribution to the effective action (here we move to the euclidean signature)

$$S \sim \int_\Sigma d^2x\, e^{2\phi_0}n^2\,, \qquad n \in \mathbb{Z}\,. \tag{3.2}$$

To minimize the action, we must have $\phi_0 \sim -\infty$ (or $y = 0$) if there is non-zero electric flux. On the contrary, $\phi_0$ is arbitrary if there is no electric flux. Hence we come to the conclusion that the string is forced to live near the source NS5-branes if there is non-zero electric flux on the worldsheet.

In the present situation, the NS5-branes are located at the origin, $y = 0$. If we T-dualize the system, the parametrization of the radial direction is reversed, namely, $y = 0$ and $y = \infty$ are interchanged.

We would like to see the correspondence between the effective action of Matrix string theory and the action of superstring on $AdS_3 \times S^3$ in some detail. To do this, we shall first move to the euclidean worldsheet. As was obtained in the previous section, the effective field theory has the following "Liouville part",

$$S = \frac{1}{8\pi}\int_\Sigma d^2x\Big(\partial\varphi\,\bar\partial\varphi + e^{-\sqrt{(2/k)}\,\varphi} f_{\mu\nu}f_{\mu\nu}\Big)\,, \tag{3.3}$$

where $\varphi = \sqrt{2k}\phi$ up to a constant shift. To ensure the conformal invariance, we must add the background charge term,

$$\frac{Q}{8\pi}\int_\Sigma \varphi R_{(2)}\,,$$

to the above action. The background charge $Q$ must be determined such that the term $e^{-\sqrt{(2/k)}\,\varphi} f_{\mu\nu}f_{\mu\nu}$ has the correct conformal dimension. Then the effective action is given by

$$S = \big(\text{super Liouville theory with the background charge } Q\big) + \tag{3.4}$$

$$+\,\big(\mathrm{SU}(2)\text{ SWZW with level } k\big) + \big(\text{SCFT on } R^4\big)\,. \tag{3.5}$$

This theory has the central charge

$$c_{\text{total}} = \Big(1 + 3Q^2 + \frac{1}{2}\Big) + \Big(\frac{3(k-2)}{k} + \frac{3}{2}\Big) + 6\,. \tag{3.6}$$

Below we show that one must choose $Q$ differently for long and short strings. The observation of [12] that the CFT on the short and long strings describe respectively the Coulomb branch and the Higgs branch near the small instanton singularity.

### 3.1 Short string ($n = 0$)

In this sector it is appropriate to assign the canonical dimension 0 to the gauge field $A_\mu$. The background charge is fixed by this condition as $Q = -\sqrt{2/k}$. The central charge becomes $c = 12$.

It is easy to see the correspondence of this CFT and the world-sheet theory of short string on $AdS_3$ background. The worldsheet action of a fundamental string on $AdS_3$ is given by [2]

$$S = \frac{1}{8\pi}\int_\Sigma d^2x\Big(\partial\varphi\,\bar\partial\varphi + e^{\sqrt{(2/k)}\,\varphi}\bar\partial\gamma\,\partial\bar\gamma\Big), \tag{3.7}$$

or equivalently, by introducing the auxiliary fields $\beta$ and $\bar\beta$ we have

$$S = \frac{1}{8\pi}\int_\Sigma d^2x\Big(\partial\varphi\,\bar\partial\varphi - e^{-\sqrt{(2/k)}\,\varphi}\beta\bar\beta + \beta\,\bar\partial\gamma + \bar\beta\,\partial\bar\gamma\Big). \tag{3.8}$$

The short string theory on $AdS_3$ is defined on an arbitrary compact Riemann surface, because it should describe arbitrary propagation and interaction of closed strings in bulk. The $(\beta,\gamma)$-system should have the conformal dimensions $(1,0)$ as in Wakimoto representation [23]. From these facts we see the operator $\oint \gamma^{-1}\partial\gamma$ cannot take non-zero classical value. The RNS superstring action on $AdS_3$ background is obtained as the supersymmetric extension of the above action. In quantizing the system, we can choose the light-cone gauge $\gamma \sim z$, $\psi^+ = 0$ ($z$ is a holomorphic coordinate on $\Sigma$ and $\psi^+$ is the fermionic coordinate along the longitudinal direction) of [8, 12] and eliminate the longitudinal degrees of freedom. After this gauge fixing, we find that the CFT of the transversal degrees of freedom coincides with the one we have obtained from the gauge theory analysis, including the value of the background charge of the Liouville part.

Here we make an important remark. The gauge condition $\gamma \sim z$ is quite different from the non-zero classical value of $\oint \gamma^{-1}\partial\gamma$. One must not confuse these two distinct notions. If we impose the light-cone gauge condition on a string theory defined on a generic Riemann surface, we should impose it on each of the *local* coordinate patches. Even in flat background the light-cone gauge cannot be imposed globally unless we take a cylindrical worldsheet.

### 3.2 Long string ($n \neq 0$)

As we have seen above, in the case of $n \neq 0$, the worldsheet of the string must be near the NS5-branes, $y \sim 0$. The non-zero electric flux $\Pi \equiv e^{-\sqrt{(2/k)}\varphi}E$ also indicates that the gauge field $A_z$ and $\bar{A}_{\bar{z}}$ should have the "geometric dimension" $(1,0)$ and $(0,1)$, and the operator $e^{-\sqrt{(2/k)}\varphi}$ should have the conformal weight $(-1,-1)$. This leads to the Liouville background charge $Q = \sqrt{2k} - \sqrt{2/k}$. The central charge now becomes $c = 6k$.

This sector is identical to the CFT on the "long string" in the light-cone gauge, which is discussed in [8, 12]. As was discussed in these papers, we can bosonize the eight free fermions in this theory to define eight spin fields, which are fermions in the sense of the worldsheet as well. Using these spin fields we can construct a $\mathcal{N} = (4,4)$ superconformal algebra with $c = 6k$. It is the sum of the two superconformal algebras arising from the $\mathbf{R}^4$ part and the remaining part, with $c = 6$ and $c = 6(k-1)$, respectively.

Now we would like to compare our CFT with the CFT on the long string worldsheet. Recall that Matrix string theory describes the IIA strings and the (short or long) strings on $AdS_3 \times S^3$ are the objects in IIB string theory. So as to make a comparison we have to apply T-duality to the two-dimensional field theory.

Let us start from IIB side. The relevant part of the world-sheet action is given by eq. (3.7), which we rewrite with $\gamma = \gamma^0 + i\gamma^1$. Of course $\gamma^0$, $\gamma^1$ are the time and spatial coordinates parameterizing the boundary of euclidean $AdS_3$. We can partially gauge-fix the conformal symmetry by the condition $\gamma^0 = t$, and obtain

$$S = \frac{1}{8\pi}\int_\Sigma d^2x \Big\{ \partial\varphi\,\bar\partial\varphi + e^{\sqrt{(2/k)}\,\varphi}\Big( \big(\partial_t\gamma^1\big)^2 + \big(\partial_x\gamma^1\big)^2 + 2\,\partial_x\gamma^1 \Big)\Big\}, \tag{3.9}$$

We can T-dualize this action according to the standard procedure(see, for example, [24]), with respect to the U(1) isometry $\gamma^1 \longrightarrow \gamma^1 + \alpha$

$$S = \frac{1}{8\pi}\int_\Sigma d^2x \Big\{ \partial\varphi\,\bar\partial\varphi + e^{-\sqrt{(2/k)}\,\varphi}\Big( \big(\partial_t\tilde\gamma^1\big)^2 + \big(\partial_x\tilde\gamma^1\big)^2 \Big) + 2i\,\partial_t\tilde\gamma^1 \Big\}, \tag{3.10}$$

where $\tilde\gamma^1$ denotes the dual coordinate of $\gamma^1$.

On the other hand, our effective action of Matrix string theory eq. (3.3) can be rewritten, if a suitable source term ensuring non-zero electric flux on the worldsheet is added, as follows

$$\begin{aligned} S &= \frac{1}{8\pi}\int_\Sigma d^2x \left\{ \partial\varphi\,\bar\partial\varphi + e^{-\sqrt{(2/k)}\,\varphi}\Big( \big(\partial_t A_1\big)^2 + \big(\partial_x A_1\big)^2 \Big)\right\} + \\ &\quad + 2i\left( \oint_{t=\infty} - \oint_{t=-\infty} A_1\,dx \right) = \\ &= \frac{1}{8\pi}\int_\Sigma d^2x \left\{ \partial\varphi\,\bar\partial\varphi + e^{-\sqrt{(2/k)}\,\varphi}\Big( \big(\partial_t A_1\big)^2 + \big(\partial_x A_1\big)^2 \Big) + 2i\,\partial_t A_1 \right\}, \end{aligned} \tag{3.11}$$

where we have chosen the $A_0 = 0$ gauge, and added a gauge-fixing term with respect to the residual gauge symmetry. Action eq. (3.11) is clearly the same as eq. (3.10) if we make the identification $\tilde\gamma^1 = A_1$.

Some comments are in order. First of all, long strings have non-zero winding number $\langle \oint \gamma^{-1} d\gamma \rangle = 1$, which implies that $\gamma$ should be of conformal dimension $-1$. This assignment of conformal weight forces us to impose the condition $\gamma \sim z$ *globally*. This is possible only when the worldsheet is cylindrical. Hence long strings must have cylindrical worldsheets, as was mentioned previously. Note also that this assignment of conformal weight is consistent with the interpretation that $\gamma$ and $A_\mu$ are dual to each other.

Moreover, the term $\sim 2e^{\sqrt{(2/k)}\,\varphi}\partial_x\gamma$ in eq. (3.9) plays the role of source term for the winding number of $\gamma$, while the corresponding term $\sim 2i\,\partial_t A_1$ in the dual action eq. (3.11) is no other than the source term for the electric field $\Pi \equiv e^{-\sqrt{(2/k)}\,\varphi} E$. Therefore we can say that the non-zero winding number of $\gamma$ in IIB theory side precisely corresponds to non-zero electric flux in the IIA Matrix string theory side.

In the above argument we can see explicitly, as stated previously, the interchange of $y = 0$ and $y = \infty$ under the T-duality transformation. Hence a long string, which is forced to live near $y \sim 0$ in IIA picture, is at $y \sim +\infty$ in IIB picture.

The argument given above was based on a proposal of [22] that we should assign the canonical dimension 0 to the gauge field $A_\mu$ in the Coulomb branch CFT, while we should assign the geometrical dimension 1 in the Higgs branch. Although originally we have been analyzing the Coulomb branch, the non-zero electric flux requires the redefinition of the conformal dimension of the gauge field, which then leads us to the physics of the Higgs branch. At the same time the object becomes a long string, whose dynamics suitably describes the Higgs branch CFT near the small instanton singularity, as was discussed in [12].

## 4. Twisted sectors

Matrix string theory contains various sectors in which some fundamental strings are glued together. The field contents on the worldsheet of glued strings are the same as those of a single string, so we can determine whether a glued string is long or short in the same way as with a single string. A generic sector of Matrix string theory corresponds to a set of glued strings, each of which has an arbitrary length. Some of them are long, i.e., there is non-zero electric flux on their worldsheet. The sum of the length of the long strings is the NS1-brane charge $p$, which is fixed. By the argument of the charge conservation, two strings can be glued together only when there is equal electric flux on the two worldsheets [21]. In particular, a long string and a short string cannot be glued together. Then we are led to the following conformal field theory for this system: $\mathrm{Sym}^{N-p}(\mathcal{M}_{\mathrm{short}}) \times \mathrm{Sym}^{p}(\mathcal{M}_{\mathrm{long}})$, where $\mathcal{M}_{\mathrm{short}}(\mathcal{M}_{\mathrm{long}})$ stands for the theory of a single short(long) string, as described in the previous section. Here we mean by $\mathrm{Sym}^{N}(\mathcal{M})$ the $N$-th symmetric product of a conformal field theory $\mathcal{M}$. This operation is defined in a similar manner as the sigma model on a symmetric orbifold $\mathcal{M}^N/S_N$ is defined from the sigma model on $\mathcal{M}$.

Matrix string theory contains various twisted sectors describing many glued strings. So we expect that all of the connected and disconnected worldsheets having arbitrary genera can be reproduced in the large $N$-limit. Moreover, we further expect that the integration over the moduli of the worldsheets corresponds precisely to the large $N$ limit of Matrix string theory. This was conjectured in [14], and more detailed analyses were given in [25] based on the interpretation of the BPS instantons in Matrix string theory as several plane curves. Although the analyses of [25] were done in the flat background, this remarkable claim may be true even in the presence of NS5-branes. In this way we might be able to recover the full second quantized IIB superstring theory on $AdS_3 \times S^3 \times R^4$ for the short string sector in the large $N$-limit.

How about the long string sector? Since the NS1-brane charge $p$ is fixed, we have at most $p$-glued long string sector. This means that we cannot consider a worldsheet of a long string with arbitrarily large genus (although we assume $p \gg 1$). Furthermore the moduli of the worldsheet should be frozen, because the worldsheet of a long string must be cylindrical, lying near the boundary of $AdS_3$ in the IIB picture. Therefore it is more appropriate to regard the CFT of the long string sector as the boundary CFT itself, rather than the theory of gravity or string theory in bulk $AdS_3$.

In our previous paper [7] we treated the first quantized RNS superstring on $AdS_3 \times S^3 \times T^4$ with the covariant formalism. We constructed a BRST invariant state that corresponds to the vacuum of the spacetime SCFT for the case of $p = 1$. As for $p > 1$ no such state was found. That observation fits nicely with the results obtained in this paper.

We can see that if there were a *single* string carrying a winding charge greater than 1, it would lead to an inconsistency. In fact, the assumption $\langle \oint \gamma^{-1} d\gamma \rangle = p$ or $\gamma \sim z^p$ implies that $(\beta, \gamma)$ have conformal weights $(p+1, -p)$, hence we have to improve the stress tensor as follows

$$T_{\text{total}} = T_{\text{worldsheet}} + p\, \partial J^3 \,. \tag{4.1}$$

This stress tensor would give rise to the central charge $c = 6kp^2$, if the worldsheet theory were originally a critical string theory. Hence this naive argument cannot explain the correct central charge unless $|p| = 1$.

In addition, our previous arguments have shown that the shift in the conformal weight of the gauge field, $A$, due to the electric flux is always 1, *no matter how large the flux is.* In the IIB picture the conformal weight of $\gamma$ is always $-1$ when there is a non-zero electric flux in IIA picture, regardless of how large it is. So the winding number of a single long string is always 1.

So, the winding charge $p > 1$ must be carried by some glued long strings. They must be composed of precisely $p$ single long strings, which leads to the CFT of $\mathrm{Sym}^p(\mathcal{M}_{\text{long}})$ with the correct central charge $c = 6kp$. We believe that the Matrix string theory is the most natural framework to explain why such an orbifold CFT arises.

## 5. Spectrum in the long string sector

In this section we discuss the spectrum of chiral primaries in the long string sector. As was given in the previous section, the CFT of the long string sector is given by $\mathrm{Sym}^p(\mathcal{M}_{\text{long}})$, where $\mathcal{M}_{\text{long}}$ is the CFT of a single long string. $\mathcal{M}_{\text{long}}$ is the product of three CFT's, namely, a super Liouville theory with $Q = \sqrt{2k} - \sqrt{(2/k)}$, an SWZW theory with level $k$ and the superconformal sigma model on $M^4$.[2] The CFT on $M^4$ and the remaining part have $\mathcal{N} = (4,4)$ superconformal algebras with the central charge $c = 6$ and $c = 6(k-1)$, respectively. According to this product structure,

[2]Previously we assumed that $M^4 = R^4$. In this section we shall somehow generalize the situation and consider $M^4$ as one of $R^4, T^4$ or $K3$. Of course, as we have mentioned previously, if we consider the generalization for the choice of the internal manifold at the stage of Matrix string theory, there must arise the subtleties concerning the compactification of Matrix theory on higher-dimensional torus. But in this section we shall forget these difficulties and simply replace $R^4$ by $T^4$ or $K3$.

any chiral primary of $\mathcal{M}_{\rm long}$ can be written as the product of a chiral primary of $M^4$ part and the chiral primary of the remaining part. We denote them as follows:

$$\mathcal{O}(\omega, l) = \mathcal{O}_\omega\, \mathcal{O}_l, \quad \left(\forall \omega \in H^*(M^4),\ l = 0, \dots, k-2\right). \tag{5.1}$$

Here $\mathcal{O}_\omega$ is one of the chiral primaries of the $M^4$ part. They are labeled by the cohomology element, $\omega$, of $M^4$. $\mathcal{O}_l$ is a chiral primary of the remaining part, which has the following form [12, 26];

$$\mathcal{O}_l \stackrel{\rm def}{=} e^{l\sqrt{(2/k)}\,\varphi} V_l\,, \quad (l = 0, \dots, k-2)\,. \tag{5.2}$$

where $V_l$ is the highest weight operator in bosonic $\mathrm{SU}(2)_{k-2}$ WZW theory with spin $l$, characterized by the following OPE with the bosonic SU(2) currents

$$k^3(z)\, V_l(0) \sim \frac{l}{2z}\, V_l(0)\,, \qquad k^+(z)\, V_l(0) \sim 0\,. \tag{5.3}$$

The chiral primary $\mathcal{O}(\omega, l)$ has the following quantum numbers

$$h = j = \frac{q(\omega) + l}{2}\,, \qquad \bar{h} = \bar{j} = \frac{\bar{q}(\omega) + l}{2}\,, \quad \left(\omega \in H^{q(\omega)\,\bar{q}(\omega)}(M^4)\right). \tag{5.4}$$

Alternatively, we can consider the corresponding Ramond vacuum $|\omega, l\rangle$ that is obtained by spectral flow:

$$\begin{aligned} L_0|\omega, l\rangle &= \frac{k}{4}|\omega, l\rangle\,, \qquad K_0^3|\omega, l\rangle = \left(\frac{q(\omega)}{2} + \frac{l}{2} - \frac{k}{2}\right)|\omega, l\rangle\,, \\ [3pt]\bar{L}_0|\omega, l\rangle &= \frac{k}{4}|\omega, l\rangle\,, \qquad \bar{K}_0^3|\omega, l\rangle = \left(\frac{\bar{q}(\omega)}{2} + \frac{l}{2} - \frac{k}{2}\right)|\omega, l\rangle\,. \end{aligned} \tag{5.5}$$

Now let us turn to the analysis for whole long string sector, $\mathrm{Sym}^p(\mathcal{M}_{\rm long})$. First of all, the spectrum for the untwisted sector is essentially the same as that for the single long string $\mathcal{M}_{\rm long}$. We again exhibit it as the spectrum of Ramond vacua:

$$\begin{aligned} L_0|\omega, l; (0)\rangle &= \frac{k}{4}|\omega, l; (0)\rangle\,, \qquad K_0^3|\omega, l; (0)\rangle = \left(\frac{q(\omega)}{2} + \frac{l}{2} - \frac{k}{2}\right)|\omega, l; (0)\rangle\,, \\ [3pt]\bar{L}_0|\omega, l; (0)\rangle &= \frac{k}{4}|\omega, l; (0)\rangle\,, \qquad \bar{K}_0^3|\omega, l; (0)\rangle = \left(\frac{\bar{q}(\omega)}{2} + \frac{l}{2} - \frac{k}{2}\right)|\omega, l; (0)\rangle\,. \end{aligned} \tag{5.6}$$

The analyses for the twisted sectors are more difficult. In general they are classified by Young tableaus $(n_1, \dots, n_s)$ composed of $p$ boxes $(n_1 \geq \cdots \geq n_s > 0$, $\sum_{i=1}^s n_i = p)$. The sector labeled by a tableau $(n_1, \dots, n_s)$ can be decomposed into a set of $\mathrm{Z}_{n_i}$-twisted sectors. In other words it can be regarded as composed of $s$ glued strings of length $(n_1, \dots, n_s)$. If we are interested in single particle states, we only have to consider the $\mathrm{Z}_m$-twisted sector, or the sector of a single glued string of length $m$.

The superconformal algebra (SCA) suitably acting on the Hilbert space of the $\mathrm{Z}_m$-twisted sector is constructed in the following manner (see for example [27]): First we make up the SCA $\left\{L'_n,\ G'^{\alpha a}_n,\ K'^{\alpha\beta}_n\right\}$ of the glued string variables as in the case of the single long string $\mathcal{M}_{\mathrm{long}}$ with $c=6k$. Second, we must mod out by $\mathrm{Z}_m$-action, and get the desired SCA $\hat{\mathcal{A}}$ with $c=6mk$. More explicitly, we should define the superconformal generators (in the Ramond sector) for the $\mathrm{Z}_m$-twisted sector as follows[3]

$$\hat{L}_n \stackrel{\mathrm{def}}{=} \frac{1}{m}L'_{mn}+\frac{k}{4}\left(m-\frac{1}{m}\right)\delta_{n0}\,,\qquad \hat{K}^{\alpha\beta}_n \stackrel{\mathrm{def}}{=} K^{\alpha\beta\,\prime}_{mn}\,,$$
$$\hat{G}^{\alpha a}_n \stackrel{\mathrm{def}}{=} \frac{1}{\sqrt{m}}G^{\alpha a\,\prime}_{mn}\,. \tag{5.7}$$

In this expression for $\hat{L}_n$ the second term of the r.h.s. corresponds to the contribution from the schwarzian derivative of the covering map $w=z^m$.

We can now present the complete spectrum of chiral primaries for the $\mathrm{Z}_m$-twisted sector. The Ramond vacua in this sector should have the same spectrum of weight and $R$-charge as those in the CFT of a single long string

$$\begin{aligned}
L'_0|\omega,l;(m)\rangle &= \frac{k}{4}|\omega,l;(m)\rangle\,, \qquad K^{3\,\prime}_0|\omega,l;(m)\rangle=\left(\frac{q(\omega)}{2}+\frac{l}{2}-\frac{k}{2}\right)|\omega,l;(m)\rangle\,,\\
\bar{L}'_0|\omega,l;(m)\rangle &= \frac{k}{4}|\omega,l;(m)\rangle\,, \qquad \bar{K}^{3\,\prime}_0|\omega,l;(m)\rangle=\left(\frac{\bar{q}(\omega)}{2}+\frac{l}{2}-\frac{k}{2}\right)|\omega,l;(m)\rangle\,.
\end{aligned} \tag{5.8}$$

Hence, by the definitions eq. (5.7) we obtain the following relations

$$\begin{aligned}
\hat{L}_0|\omega,l;(m)\rangle &= \left\{\frac{k}{4m}+\left(\frac{km}{4}-\frac{k}{4m}\right)\right\}|\omega,l;(m)\rangle=\frac{km}{4}|\omega,l;(m)\rangle\,,\\
\hat{K}^3_0|\omega,l;(m)\rangle &= \left(\frac{q(\omega)}{2}+\frac{l}{2}-\frac{k}{2}\right)|\omega,l;(m)\rangle\,,\\
\bar{\hat{L}}_0|\omega,l;(m)\rangle &= \frac{km}{4}|\omega,l;(m)\rangle\,, \qquad \bar{\hat{K}}{}^3_0|\omega,l;(m)\rangle=\left(\frac{\bar{q}(\omega)}{2}+\frac{l}{2}-\frac{k}{2}\right)|\omega,l;(m)\rangle\,.
\end{aligned} \tag{5.9}$$

Finally we translate the above relations in the Ramond sector to those in NS sector by means of the spectral flow. As a result, the chiral primaries in $\mathrm{Z}_m$-twisted sector

[3]The situation becomes more complicated for the SCA of NS sector, since we must define them separately according to whether $m$ is even or odd. This is the main reason why here we work in the Ramond sector and make use of the spectral flow instead of dealing with the NS sector directly.

$\mathcal{O}(\omega, l; (m))$ have the following quantum numbers

$$h = j = \frac{q(\omega) + l + k(m-1)}{2}\,,$$
$$\bar{h} = \bar{j} = \frac{\bar{q}(\omega) + l + k(m-1)}{2}\,,$$
$$(l = 0, \ldots, k-2, m = 1, \ldots, p)\,. \tag{5.10}$$

In this way we can reproduce almost all the chiral primaries of single particle type that are expected from the correspondence with the SCFT on $\mathrm{Sym}^{pk}(M^4)$,*except for the absence of the following sequence of states*:

$$h = j = \frac{q(\omega) + km - 1}{2}\,, \qquad \bar{h} = \bar{j} = \frac{\bar{q}(\omega) + km - 1}{2}\,, \quad (m = 1, \ldots, p)\,. \tag{5.11}$$

Notice that the case $m = 1$ in (5.11) is no other than the "first missing state" discussed in [12]. There it was discussed that the absence of this state is related to the small-instanton singularity of the moduli space of $D1/D5$-brane bound states. We find its cousins in the $\mathrm{Z}_m$-twisted sectors $m > 1$. Although we still have the missing states in the framework of Matrix string theory, there are "not too many" of them. Especially, the bound for the R-charge $\sim pk/2$ expected from the relation to $\mathrm{Sym}^{pk}(M^4)$ sigma model is correctly reproduced.

In [12] it was also pointed out that, in the single long string theory, the continuous spectrum appears above the threshold conformal weight, $\Delta_0 = (k-1)^2/4k \sim k/4$, which would make the counting of the chiral primaries above the threshold difficult. This result is based on the study of Liouville theory [28], in which it is claimed that we have two different types of states in quantum Liouville theory: one of them is a non-normalizable state which form a discrete spectrum, and the other is a normalizable state which form a continuous spectrum. The latter appears above the threshold, $\Delta_0 \equiv Q^2/8$, where $Q$ is the background charge of Liouville theory. Here we shall show that the threshold value also becomes $p$-times larger when we consider the CFT of $p$ long strings, $\mathrm{Sym}^p(\mathcal{M}_{\mathrm{long}})$.

It is easy to see this in the untwisted sector. We only have to find the normalizable state with the smallest conformal weight in this sector. Obviously this state has the form $|\Delta_0\rangle^{\otimes p}$, where $|\Delta_0\rangle$ is the normalizable state with the lowest energy in the single long string theory

$$L_0 |\Delta_0\rangle \sim \frac{k}{4} |\Delta_0\rangle\,. \tag{5.12}$$

Therefore the threshold for the untwisted sector is equal to $\hat{\Delta}_0 \sim pk/4$.

In order to calculate the threshold value for general twisted sectors we have to recall the definition of the Virasoro operator in the $\mathrm{Z}_m$-twisted sector (5.7). Then the threshold for the $Z_m$-twisted sector becomes

$$\hat{\Delta}_0 \sim \frac{k}{4m} + \frac{k}{4}\left(m - \frac{1}{m}\right) = \frac{mk}{4}\,. \tag{5.13}$$

From these results we can conclude that the threshold value is the same for all the twisted sectors characterized by arbitrary Young tableaus $(n_1, \ldots, n_s) \in Y_p$

$$\hat{\Delta}_0 = \sum_{i=1}^{s} \frac{n_i k}{4} = \frac{pk}{4}\,. \tag{5.14}$$

We make one comment. It is known that the spectrum of chiral primaries in $\mathrm{Sym}^{pk}(M^4)$ sigma model agrees with that of supergravity below the bound $\sim pk/4$. This claim was first proved in the case of $M^4 = K3$ by analyzing the elliptic genus [29], and more recently it has been proved in the case of $M^4 = T^4$ by analyzing the "new SUSY index" [30]. The value of the upper bound is the same, $pk/4$. Although apparently the origins of the bound are different between the analyses of [29, 30] and ours, there might be some relationship between the two. This is because supergravity on an $AdS_3 \times S^3 \times M^4$ background has only the discrete spectrum, so that the emergence of continuous spectrum may lead to a failure in the correspondence between the description of supergravity and the boundary SCFT.

## 6. Conclusions and discussions

In this paper we studied the multi-string system on $AdS_3 \times S^3 \times M^4$ in the framework of Matrix string theory. Among other things, we have successfully presented a unified framework of the various short and long string sectors. Although gauge field in two dimensions has no local physical degrees of freedom, it plays an extremely important role, namely, the VEV of electric field on the worldsheet distinguishes between the short and long string sectors.

Now we would like to mention some questions which could be discussed in future works.

In section 3 we have shown the T-duality between the Matrix string action and the $AdS_3 \times S^3$ string action, as far as their bosonic parts are concerned. It would be interesting to investigate further and see the full T-duality. Of course, some fermions are missing on the gauge-theory side (which would correspond to the superpartner of bosonic $\mathrm{SL}(2,R)$ currents on the superstring theory.) Perhaps this aspect will be better understood when we come to a deeper understanding of the relation between the U(1) gauge symmetry of Matrix string theory and the reparametrization invariance of the superstring theory. To this aim the formulation of Matrix theory in a manifestly covariant manner may be useful. Some approaches to this subject were given in [31].



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In section 5 we discussed the problem of the missing chiral primaries by the analysis of the CFT of $p$ long strings. Although we still have some states missing, as was pointed out in [12], the correspondence with the $\mathrm{Sym}^{pk}(M^4)$ sigma model is not "too bad". In particular, by taking account of the various glued strings we have reproduced the bound $\sim pk/2$ which is precisely the same as is expected from the analysis of the $\mathrm{Sym}^{pk}(M^4)$ sigma model.

However, there seems to be a further subtlety regarding the spectrum of the multi-particle chiral primaries, for our construction seems to yield the spectrum interpretable as the cohomology of $\mathrm{Sym}^p(\mathrm{Sym}^k(M^4))$, not of $\mathrm{Sym}^{pk}(M^4)$. In other words there should be an U-duality transformation which changes the value of $k$ and $p$, while preserving their product. At present it is not clear whether our construction in the framework of Matrix string theory has this duality symmetry. In any case, we will need further study to get a complete understanding of this issue.

The problem of the threshold for the continuous spectrum is also interesting. We have shown that our analysis of the CFT of $p$ long strings yields a threshold $p$-times larger, $\sim kp/4$. This is the same as the threshold obtained in [29, 30]. It was discussed there that the correspondence between the supergravity in bulk and the boundary SCFT fails above this threshold. Perhaps it may be interesting to study the relation between this issue and the physics of three-dimensional black holes, as the above threshold coincides with the energy at which the first excited BTZ black hole (the massless BTZ black-hole) appears.

## Acknowledgments

We would like to thank T. Eguchi for discussions and useful comments. We would also like to thank B.D. Bates for careful reading of the manuscript. K.H is supported in part by JSPS Research Fellowships, and also Y.S. is supported in part by the Grant-in-Aid from the Ministry of Education, Science and Culture, Priority Area: "Supersymmetry and Unified Theory of Elementary Particles" (♯707).

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